

# Interval Estimation of a Population Proportion

## ■ Interval Estimate

$$\bar{p} \pm z_{\alpha/2} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

- where
- 1 -  $\alpha$  is the confidence coefficient
  - $z_{\alpha/2}$  is the z value providing an area of  $\alpha/2$  in the upper tail of the standard normal probability distribution
  - $\bar{p}$  is the sample proportion

# Example: Red Sea, Inc.

## ■ Interval Estimation of a Population Proportion

If in a sample of 500 tourists, 220 plan to repeat their trips the following year. Calculate the 95% confidence interval of travelers who expect to repeat their trips.

# Example: Red Sea, Inc.

## ■ Interval Estimate of a Population Proportion

$$\bar{p} \pm z_{\alpha/2} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

where  $n = 500$ ,  $p = 220/500 = .44$ ,  $z_{\alpha/2} = 1.96$

$$.44 \pm 1.96 \sqrt{\frac{.44(1-.44)}{500}}$$

$$.44 \pm .0435$$

RSI is 95% confident that the proportion of travelers who expect to repeat their trips is between .3965 and .4835.

# Sample Size for an Interval Estimate of a Population Proportion

- Let  $E$  = the maximum sampling error mentioned in the precision statement.
- We have

$$E = z_{\alpha/2} \sqrt{\frac{p(1-p)}{n}}$$

- Solving for  $n$  we have

$$n = \frac{(z_{\alpha/2})^2 p(1-p)}{E^2}$$

# Example: Political Science, Inc.

## ■ Sample Size for an Interval Estimate of a Population Proportion

Suppose that RSI would like a .99 probability that the sample proportion is within + .03 of the population proportion.

How large a sample size is needed to meet the required precision?

# Example: Political Science, Inc.

## ■ Sample Size for Interval Estimate of a Population Proportion

At 99% confidence,  $z_{.005} = 2.576$ .

$$n = \frac{(z_{\alpha/2})^2 \underline{p}(1 - \underline{p})}{\underline{E}^2} = \frac{(2.576)^2 (.44)(.56)}{(.03)^2} \cong 1817$$

**Note:** We used .44 as the best estimate of  $\pi$  in the above expression. If no information is available about  $\pi$ , then .5 is often assumed because it provides the highest possible sample size. If we had used  $\pi = .5$ , the recommended  $n$  would have been 1843.

# Estimating population proportion in MINITAB

## ■ Stat > Basic Statistics > 1 Proportion

### Dialog box items

- **Samples in columns:** Choose if you have data in columns, then enter the columns containing the sample data. Each cell of these columns must be one of two possible values and correspond to one item or subject. The possible values in the columns must be identical if you enter multiple columns.
- **Summarized data:** Choose if you have summary values for the number of trials and successes.
  - **Number of trials:** Enter a single value for the number of trials. Often the number of trials will be your sample size.
  - **Number of events:** Enter the number of observed events. You may enter more than one value.