

# Goodness of Fit Test

- A Hypothesis Test for Proportions of a Multinomial Population

# Hypothesis (Goodness of Fit) Test for Proportions of a Multinomial Population

1. Set up the null and alternative hypotheses.
2. Select a random sample and record the observed frequency,  $f_i$ , for each of the  $k$  categories.
3. Assuming  $H_0$  is true, compute the expected frequency,  $e_i$ , in each category by multiplying the category probability by the sample size.

# Hypothesis (Goodness of Fit) Test for Proportions of a Multinomial Population

4. Compute the value of the test statistic.

$$\chi^2 = \sum_{i=1}^k \frac{(f_i - e_i)^2}{e_i}$$

5. Reject  $H_0$  if p-value =  $P(\chi^2_{k-1} > \chi^2_{cal}) < \alpha$  (where  $\alpha$  is the significance level and there are  $k - 1$  degrees of freedom).

## Example: ABC Homes

### ■ Multinomial Distribution Goodness of Fit Test

ABC Homes manufactures four models of prefabricated homes, a two-story colonial, a ranch, a split-level, and an A-frame. To help in production planning, management would like to determine if previous customer purchases indicate that there is a preference in the style selected.

## Example: ABC Homes

### ■ Multinomial Distribution Goodness of Fit Test

The number of homes sold of each model for 100 sales over the past two years is shown below.

<u>Model</u>	<u>Colonial</u>	<u>Ranch</u>	<u>Split-Level</u>	<u>A-Frame</u>
# Sold	30	20	35	15

## Example: ABC Homes

### ■ Multinomial Distribution Goodness of Fit Test

Let:

$\pi_C$  = population proportion that purchase a colonial

$\pi_R$  = population proportion that purchase a ranch

$\pi_S$  = population proportion that purchase a split-level

$\pi_A$  = population proportion that purchase an A-frame

## Example: ABC Homes

### ■ Multinomial Distribution Goodness of Fit Test

- Hypotheses

$$H_0: \pi_C = \pi_R = \pi_S = \pi_A = .25$$

$H_a$ : The population proportions are not all equal

## Example: ABC Homes

### ■ Multinomial Distribution Goodness of Fit Test

- Expected Frequencies

$$e_1 = .25(100) = 25$$

$$e_2 = .25(100) = 25$$

$$e_3 = .25(100) = 25$$

$$e_4 = .25(100) = 25$$

- Test Statistic

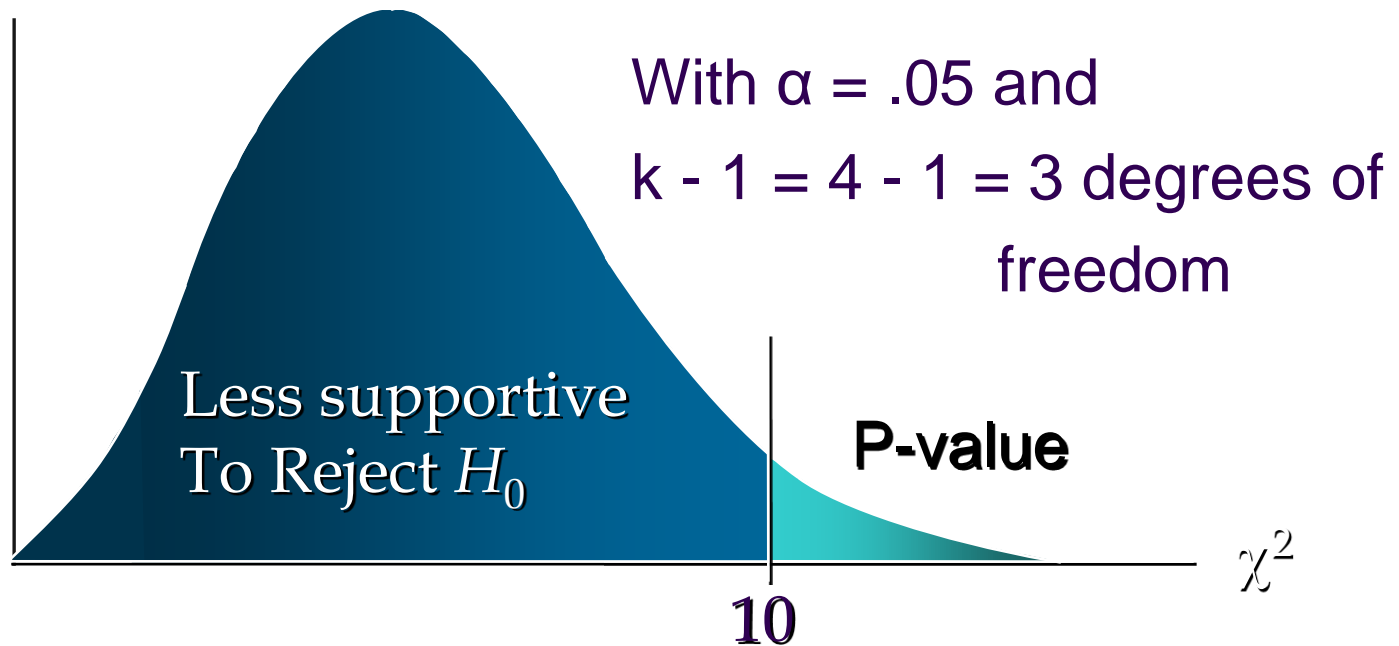
$$\chi^2 = \frac{(30-25)^2}{25} + \frac{(20-25)^2}{25} + \frac{(35-25)^2}{25} + \frac{(15-25)^2}{25}$$

$$= 1 + 1 + 4 + 4$$

$$= 10$$

## Example: ABC Homes

- Multinomial Distribution Goodness of Fit Test
  - p- value



## Example: ABC Homes

### ■ Multinomial Distribution Goodness of Fit Test

#### • Conclusion

p-value =  $P(\chi^2_{df=3} > 10) = 0.0186 < 0.05 = \alpha$ . We reject the assumption there is no home style preference, at the .05 level of significance.